

# 4/ECO-251 Syllabus-2023

**2026**

( May-June )

**FYUP : 4th Semester Examination**

**ECONOMICS**

**( Mathematical Methods for Economics II )**

( ECO-251 )

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks for the questions.*

Answer **five** questions, taking at least **one** from each Unit.

## **UNIT—I**

1. (a) What is a vector space? What are the axioms that define a vector space? 2+2=4

(b) For scalars  $\alpha=3$ ,  $\beta=4$  and vectors  $u=(1, 2)$  and  $v=(3, 4)$ , verify the following property : 3

$$\alpha(u + v) = \alpha u + \alpha v$$

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- (c) What are the conditions for non-singularity of a matrix? Test whether the following matrix is non-singular :  $1+3=4$

$$A = \begin{bmatrix} 7 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6 \end{bmatrix}$$

- (d) If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

prove that  $|A \cdot B| = |A| \cdot |B|$ . 4

2. (a) Show without expansion that

$$\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix} = \begin{bmatrix} c & b & a \\ b & a & c \\ a & c & b \end{bmatrix} \quad 3$$

- (b) If

$$[x \ 2 \ -2] \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ 2 \\ -2 \end{bmatrix} = 0$$

find  $x$ . 4

- (c) If

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 3 & 1 & -2 \\ -1 & 0 & 4 \end{bmatrix}$$

prove that  $A^{-1} \cdot A = I$ . 8

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3. (a) Solve the following system of simultaneous equations using Cramer's rule : 8

$$\begin{aligned} 2x_1 + 3x_2 - x_3 &= 15 \\ 4x_2 + 2x_3 &= 16 \\ 3x_1 + 2x_2 &= 18 \end{aligned}$$

- (b) Define rank of a matrix. 2

- (c) Find the rank of the matrix  $A$

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 4 & -3 & 4 \\ 2 & 4 & -4 \end{bmatrix} \quad 5$$

### UNIT—II

4. (a) What is a homothetic function? Are all homothetic functions homogeneous?  $3+1=4$

- (b) Show that the following functions are homogeneous of degree 1 : 4

(i)  $Q(L, K) = (K^{0.5} + L^{0.5})^2$

(ii)  $P(L, K) = A[\alpha K^{-\rho} + (1-\alpha)L^{-\rho}]^{-1/\rho}$

- (c) What is the condition for differentiability of a function at a point defined on an interval? 2

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(d) Is the function

$$f(x) = 2 - x \quad \text{when } 1 \leq x \leq 2$$

$$= x - \frac{1}{2}x^2 \quad \text{when } x > 2$$

differentiable at  $x=2$ ? 5

5. (a) Find the maximum/minimum value of the function

$$u = 2x^2 + y^2 - 4x + 8y \quad 6$$

(b) A multiproduct monopolist produces two commodities which are technically related in production. The joint cost for the firm is  $C = q_1^2 + 2q_1q_2 + q_2^2$ . The demand laws of the products are  $p_1 = 10 - q_1$  and  $p_2 = 20 - 2q_2$ . Find the equilibrium values of  $q_1$ ,  $q_2$ ,  $p_1$ ,  $p_2$  and the profit  $\pi$ . 6+3=9

6. (a) (i) What is the difference between 'constrained' and 'unconstrained' optimization? 6+3=9

(ii) State the necessary and sufficient conditions for constrained optimization. 2+3=5

(b) A firm can buy two inputs capital ( $K$ ) and labour ( $L$ ) at ₹ 18 per unit and ₹ 8 per unit respectively and faces the production function  $Q = 24K^{0.6}L^{0.3}$ . What is the maximum output it can produce for a budget of ₹ 50,000? 10

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### UNIT—III

7. (a) Distinguish between differential equation and difference equation. 3

(b) Solve the following differential equations (any four) : 3×4=12

(i)  $3xydy = (x^2 + y^2)dx$

(ii)  $y(1+x)dx + x(1-y)dy = 0$

(iii)  $(2x+3y)dx + (3x-x^2y)dy = 0$

(iv)  $\frac{dy}{dx} + e^x + ye^x = 0$

(v)  $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$

8. (a) The demand and supply functions are given by  $D_t = 5 - 3P_t$ ;  $S_t = 6 + 8P_{t-1}$ . Determine the time path of price if the initial price is ₹ 10. 6

(b) Find the solutions of the following difference equations (any three) : 3×3=9

(i)  $y_t = Ly_{t+1}$ ,  $y_0 = M$

(ii)  $y_{t+1} - 5y_t = 1$ ,  $y_0 = \frac{7}{4}$

(iii)  $y_{x+1} = 0.2y_x + 4$ ,  $y_0 = 4$

(iv)  $2y_{t+1} - y_t = 6$ ,  $y_0 = 7$

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9. (a) The marginal rate of substitution of  $Y$  for  $X$  is given by

$$\frac{-a}{\beta} \cdot \frac{y+b}{x+a}$$

Show that the form of utility function would be  $u = (x+a)^\alpha \cdot (y+b)^\beta$ , where  $a$ ,  $b$ ,  $\alpha$ ,  $\beta$  are constants.

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- (b) The demand and supply functions in a market is given by

$$Q_d = a - bp$$

$$Q_s = -c + dp$$

Obtain the time path of price  $P(t)$  if the rate of change of price over time is directly proportional to excess demand, the adjustment coefficient being  $\lambda (\lambda > 0)$  and show that it is dynamically stable.

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#### UNIT—IV

10. (a) What are the assumptions of Leontief's static model?

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- (b) Point out the limitations of input-output analysis.

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- (c) The following table gives the inter-industry transactions in millions of rupees for an economy with three sectors  $S_1, S_2, S_3$  :

	$S_1$	$S_2$	$S_3$	Final demand	Total output
$S_1$	50	25	25	100	200
$S_2$	40	50	10	200	300
$S_3$	100	50	50	300	600

Calculate the coefficient matrix.

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11. (a) The technological coefficient matrix is given by

$$A = \begin{bmatrix} 0.3 & 0.2 & 0.4 \\ 0 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

Find the output level consistent with the model if  $D_1=300$ ,  $D_2=500$ ,  $D_3=400$ .

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- (b) What are the applications of linear programming techniques in the field of economics?

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12. (a) A manufacturing unit plans to produce two products I and II which will be stored in a storage area whose capacity is 30000 square feet. Product I takes 3 square feet of the space per unit, while product II requires 4 square feet per

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unit. It takes 4 machine hours to manufacture a unit of product I and 8 machine hours are required for a unit of product II. During the production process, 48000 machine hours are available. Also 36000 man hours are available for finishing the products. A unit of product I takes 4 man hours for finishing and product II takes 3 man hours. Profit contributions are ₹ 20 per unit of product I and ₹ 60 per unit of product II.

In order to maximize profit, how many units of each product should be produced?

- (i) Formulate the linear programming problem. 7
  - (ii) Find the optimal solution graphically. 6
- (b) Why do we convert a primal problem in linear programming to a dual problem? 2

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